

Série n°2

5) Résoudre et discuter dans l'ensemble des réels $\mathbb{R}$ les équations suivantes :

$$1) \quad \frac{x-2}{x+2} - \frac{2}{x^2-4} = \frac{x+2}{x-2}$$

$$2) \quad m(2x-5) - 4x = 1 - m$$

$$3) \quad mx + 4 = m^2 + 2x$$

$$4) \quad \frac{x-1}{3} - (2x - \frac{1-x}{2}) = \frac{5}{3} - \frac{x+2}{6}$$

$$5) \quad (x-1)(x+3) = (x+1)(x-2)$$

$$6) \quad (x-2)^2 = x^2 - 5x + 1$$

$$7) \quad 2(3-2x) - 3(5x+4) = 5x - 2$$

$$8) \quad \frac{5x}{3} - \frac{1}{5} = \frac{2x}{15} + \frac{4}{3}$$

$$9) \quad \frac{-8x+6}{3} = \frac{1}{4} - \frac{2x-3}{12}$$

$$10) \quad \frac{(x+1)}{(x-2)} = \frac{(x-3)}{(x+4)}$$

$$11) \quad \frac{(x-1)}{(x-2)} = \frac{(x-3)}{(x-4)}$$

$$12) \quad \frac{(x-3)}{(x+3)} - \frac{(x+3)}{(x-3)} = \frac{3}{(x^2-9)}$$

$$13) \quad m(x-3) + x = 1 - m$$

$$14) \quad mx + 2m = m^2 + 2x$$

$$\textcircled{1)} \quad \frac{x-2}{x+2} - \frac{2}{x^2-4} = \frac{x+2}{x-2} \quad \text{et } x \in \mathbb{R} - \{2; -2\}$$

$$\Leftrightarrow \frac{x-2}{x+2} - \frac{2}{x^2-4} - \frac{x+2}{x-2} = 0$$

$$\Leftrightarrow \frac{(x-2)^2 - 2 - (x+2)^2}{(x+2)(x-2)} = 0$$

$$\Leftrightarrow (x-2)^2 - 2 - (x+2)^2 = 0$$

$$\Leftrightarrow \cancel{x^2} - \cancel{4x} + \cancel{4} - 2 - \cancel{x^2} - \cancel{4x} - \cancel{4} = 0$$

$$\Leftrightarrow -8x - 2 = 0$$

$$\Leftrightarrow -8x = +2$$

$$\Leftrightarrow x = \frac{-2}{8} = -\frac{1}{4}$$

$$\Leftrightarrow x \in \left\{ -\frac{1}{4} \right\}$$

$$\frac{a}{b} = 0$$

$$\Leftrightarrow a=0 \text{ ou } b \neq 0$$

et

$$\textcircled{2)} \quad m(2x-5) - 4x = 1 - m \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow 2mx - 4x = 5m - m + 1$$

$$\Leftrightarrow (2m - 4)x = 4m + 1$$

$$\Leftrightarrow 2(m-2)x = 4m+1 \quad \Bigg) : 2$$

$$\Leftrightarrow (m-2) \cdot x = \frac{1}{2}(4m+1)$$

$$\Leftrightarrow \begin{cases} m=2 & \text{et } 0 \cdot x = \frac{9}{2} \text{ et } x \in \emptyset \\ \text{ou} \\ m \neq 2 & \text{et } x = \frac{4m+1}{2(m-2)} \text{ et } x \in \left\{ \frac{4m+1}{2(m-2)} \right\} \end{cases}$$

$$\boxed{3)} \quad mx + 4 = m^2 + 2x \quad \text{et} \quad x \in \mathbb{R}$$

$$\Leftrightarrow mx - 2x = m^2 - 4$$

$$\Leftrightarrow (m-2) \cdot x = m^2 - 4$$

$$\Leftrightarrow \begin{cases} m=2 \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ m \neq 2 \text{ et } x = \frac{(m-2)(m+2)}{m-2} \text{ et } x \in \{m+2\} \end{cases}$$

$$-4x = 0$$

$$\Leftrightarrow x = \frac{0}{-4} = 0$$

$$\Leftrightarrow x \in \{0\}$$

$$6) (x-2)^2 = x^2 - 5x + 1 \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow \cancel{x}^2 - 4x + 4 = \cancel{x}^2 - 5x + 1$$

$$\Leftrightarrow x + 3 = 0$$

$$\Leftrightarrow x \in \{-3\}$$

$$\textcircled{8) \quad \frac{5x}{3} - \frac{1}{5} = \frac{2x}{15} + \frac{4}{3} \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow 15x - 3 = 2x + 20 \quad \downarrow \times 15$$

$$\Leftrightarrow 23x = 23$$

$$\Leftrightarrow x \in \{1\}$$

$$\textcircled{10) \quad \frac{(x+1)}{(x-2)} = \frac{(x-3)}{(x+4)} \quad \text{et} \quad x \in \mathbb{R} - \{2; -4\}$$

$$\Leftrightarrow (x+1)(x+4) = (x-2)(x-3)$$

$$\Leftrightarrow x^2 + 5x + 4 = x^2 - \cancel{4}^5 x + 6$$

$$\Leftrightarrow 0 \cdot x^2 + 10x - 2 = 0$$

$$\Leftrightarrow x = \frac{1}{5}$$

$$\Leftrightarrow x \in \left\{ \frac{1}{5} \right\}$$

Pos-it

$$\cancel{\frac{a}{b} = \frac{c}{d}} \quad \text{et } b \neq 0 \quad \text{et} \quad d \neq 0$$

$$\Leftrightarrow ad = bc$$

11) $\frac{(x-1)}{(x-2)} = \frac{(x-3)}{(x-4)}$ et $x \in \mathbb{R} - \{2; 4\}$ (Post-it)

$$\Leftrightarrow (x-1)(x-4) = (x-3)(x-2)$$

$$\Leftrightarrow x^2 - 5x + 4 = x^2 - 5x + 6$$

$$\Leftrightarrow 0 \cdot x^2 + 0 \cdot x - 2 = 0$$

$$\Leftrightarrow x \in \emptyset$$

$$\frac{a}{b} = \frac{c}{d} \quad \text{et } b \neq 0$$
$$\quad \quad \quad \text{et } d \neq 0$$

$$\Leftrightarrow ad = bc$$

12) $\frac{(x-3)}{(x+3)} - \frac{(x+3)}{(x-3)} = \frac{3}{(x^2-9)}$ et $x \in \mathbb{R} - \{-3; +3\}$

$$\Leftrightarrow \frac{(x-3)^2 - (x+3)^2}{x^2 - 9} = \frac{3}{x^2 - 9}$$

$$\Leftrightarrow \cancel{x^2} - 6x + \cancel{9} - \cancel{x^2} - 6x - \cancel{9} = 3$$

$$\Leftrightarrow -12x = 3$$

$$\Leftrightarrow x = \frac{3}{-12} = -\frac{1}{4}$$

$$\Leftrightarrow x \in \left\{-\frac{1}{4}\right\}$$

13) $m(x-3) + x = 1 - m$ et $x \in \mathbb{R}$

$$\Leftrightarrow mx + x = 1 - m + 3m$$

$$\Leftrightarrow (m+1) \cdot x = 2m+1$$

$$\Leftrightarrow \begin{cases} m = -1 \text{ et } 0 \cdot x = -1 \text{ et } x \in \emptyset \\ \text{ou} \\ m \neq -1 \text{ et } x \in \left\{ \frac{2m+1}{m+1} \right\} \end{cases}$$

13) $m(x-3) + x = 1 - m$ et $m \in \mathbb{R}$

$$\Leftrightarrow m(x-3) + m = 1-x$$

$$\Leftrightarrow (x-2) \cdot m = 1-x$$

$$\Leftrightarrow \begin{cases} x=2 & \text{et } 0 \cdot m = -1 \text{ et } m \in \emptyset \\ \text{ou} \\ x \neq 2 & \text{et } m \in \left\{ \frac{1-x}{x-2} \right\} \end{cases}$$

$$14) \quad mx + 2m = m^2 + 2x \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow mx - 2x = m^2 - 2m$$

$$\Leftrightarrow (m-2) \cdot x = m(m-2)$$

$$\Leftrightarrow \begin{cases} m \neq 2 \text{ et } x = m \text{ et } x \in \{m\} \\ \text{ou} \\ m = 2 \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R} \end{cases}$$

$$(14) \quad mx + 2m = m^2 + 2x \quad \text{et} \quad m \in \mathbb{R}$$

$$\Leftrightarrow m^2 - 2m - mx + 2x = 0$$

$$\Leftrightarrow m^2 - (x+2)m + 2x = 0$$

$$\Leftrightarrow (m-2)(m-x) = 0$$

$$\Leftrightarrow m \in \{2; x\}$$

$$\left| \begin{array}{l} \begin{cases} a+b = -x-2 \\ a \cdot b = 2x \end{cases} \\ \Leftrightarrow \begin{cases} a = -2 \\ b = -x \end{cases} \end{array} \right.$$

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15 Résoudre les équations paramétriques.

Exemple

$$\begin{aligned}
 ax + b = c & \Leftrightarrow ax = c - b \Leftrightarrow \begin{cases} a = 0 \text{ et } 0x = c - b \\ \text{ou} \\ a \neq 0 \text{ et } x = \frac{c - b}{a} \end{cases} \\
 & \Leftrightarrow \begin{cases} a = 0 \text{ et } \begin{cases} b = c \text{ et } 0x = 0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ b \neq c \text{ et } x \in \emptyset \end{cases} \\ a \neq 0 \text{ et } x \in \left\{ \frac{c - b}{a} \right\} \end{cases}
 \end{aligned}$$

- | | |
|---|---|
| 1) $(a + 1)x = 2$ | 2) $(3 + a)x = 3x - 1$ |
| 3) $(a + 2)x = 0$ | 4) $ax + 2 = b$ |
| 5) $2b + 3x = (2 + a)x$ | 6) $(m + 2)x = 3$ |
| 7) $\frac{4x + 8}{0.2} = \frac{mx}{0.8}$ | 8) $\frac{1}{2}ax - 0.36 = 0$ |
| 9) $0.45x - 0.27 = (m + 1.66)x$ | 10) $m^2(x - 1) + 3(mx - 2) = 5m - 2x$ |
| 11) $m^2(x - 1) + x = 2mx - 1$ | 12) $\frac{x - a}{a} + \frac{x - b}{b} = 0$ |
| 13) $\frac{x + a}{b} + \frac{x + b}{a} = -2$ | 14) $\frac{x + 1}{a^2} + \frac{x - 1}{b^2} = \frac{2x}{ab}$ |
| 15) $\frac{a}{b}(x - 1) + 1 = \frac{b}{a}(1 - x)$ | 16) $(2a - 1)(x - b) = a(x + b)$ |
| 17) $\frac{2}{x^2 + x} + \frac{m}{x - 1} = \frac{m}{x}$ | 18) $\frac{a}{x + 1} - \frac{b}{x - 1} = \frac{a - b}{x^2 - 1}$ |

$$\textcircled{1) \quad (a + 1)x = 2 \quad \text{et} \quad x \in \mathbb{R}}$$

$$\Leftrightarrow \begin{cases} a = -1 \text{ et } 0 \cdot x = 2 \text{ et } x \in \emptyset \\ \text{ou} \\ a \neq -1 \text{ et } x \in \left\{ \frac{2}{a+1} \right\} \end{cases}$$

$$(3) \quad (a + 2)x = 0 \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow \begin{cases} a = -2 \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ a \neq -2 \text{ et } x \in \{0\} \end{cases}$$

4)

$$ax + 2 = b \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow ax = b - 2$$

$$\Leftrightarrow \begin{cases} a = 0 & \text{et } 0 \cdot x = b - 2 \text{ et } \begin{cases} b = 2 \text{ et } 0x = 0 \\ \text{et } x \in \mathbb{R} \end{cases} \\ \text{ou} \\ a \neq 0 & \text{et } x \in \left\{ \frac{b-2}{a} \right\} \end{cases} \begin{cases} b \neq 2 \text{ et } 0 \cdot x = \underline{b-2} \\ \neq 0 \end{cases} \text{ et } x \in \emptyset$$

$$\textcircled{5) \quad 2b + 3x = (2 + a)x \quad \text{et} \quad x \in \mathbb{R}}$$

$$\Leftrightarrow 2b + 3x = 2x + ax$$

$$\Leftrightarrow 3x - ax - 2x = -2b$$

$$\Leftrightarrow (1 - a)x = -2b$$

$$\Leftrightarrow \begin{cases} a=1 & \text{et} & 0 \cdot x = -2b & \text{et} \\ \text{ou} & & & \end{cases} \begin{cases} b=0 & \text{et} & 0 \cdot x = 0 & \text{et} & x \in \mathbb{R} \\ b \neq 0 & \text{et} & x \in \emptyset \end{cases}$$

$$\begin{cases} a \neq 1 & \text{et} & x \in \left\{ \frac{-2b}{1-a} \right\} \end{cases}$$

$$\textcircled{10) \quad m^2(x-1) + 3(mx-2) = 5m-2x \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow m^2x + 3mx + 2x = m^2 + 5m + 6$$

$$\Leftrightarrow (m^2 + 3m + 2)x = (m+2)(m+3)$$

$$\Leftrightarrow (m+1)(m+2)x = (m+2)(m+3)$$

$$\Leftrightarrow \begin{cases} m = -1 \text{ et } 0 \cdot x = 2 \text{ et } x \in \emptyset \\ \text{ou} \\ m = -2 \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ (m \neq -1 \text{ et } m \neq -2) \text{ et } x \in \left\{ \frac{m+3}{m+1} \right\} \\ m \in \mathbb{R} - \{-1; -2\} \\ (m \notin \{-1; -2\}) \end{cases}$$

$$(11) \quad m^2(x-1) + x = 2mx - 1 \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow m^2x - m^2 + x = 2mx - 1$$

$$\Leftrightarrow m^2x - 2mx + x = m^2 - 1$$

$$\Leftrightarrow (m^2 - 2m + 1)x = (m-1)(m+1)$$

$$\Leftrightarrow (m-1)^2x = (m-1)(m+1)$$

$$\Leftrightarrow \begin{cases} m=1 & \text{et } 0x=0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ m \neq 1 & \text{et } x = \frac{(m-1)(m+1)}{(m-1)^2} \text{ et } x \in \left\{ \frac{m+1}{m-1} \right\} \end{cases}$$

$$\textcircled{13)} \quad \frac{x+a}{b} + \frac{x+b}{a} = -2 \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow \begin{cases} a=0 \text{ ou } b=0 \text{ et } x \in \emptyset \\ \text{ou} \\ a \neq 0 \text{ et } b \neq 0 \text{ et } \frac{ax+a^2+bx+b^2}{ab} = -\frac{2}{1} \end{cases}$$

$$\text{et } ax+a^2+bx+b^2 = -2ab$$

$$\text{et } (a+b) \cdot x = -a^2-2ab-b^2$$

$$\text{et } (a+b) \cdot x = -(a+b)^2$$

$$\text{et } \begin{cases} a = -b \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ \cancel{b = -a \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R}} \\ a \neq -b \text{ et } x \in \left\{ \begin{array}{l} -(a+b) \\ -a-b \end{array} \right\} \end{cases}$$

$$(15) \quad \frac{a}{b}(x-1) + 1 = \frac{b}{a}(1-x) \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow \begin{cases} b=0 \text{ ou } a=0 \text{ et } x \in \emptyset \\ \text{ou} \\ b \neq 0 \text{ et } a \neq 0 \text{ et } a^2(x-1) + ab = b^2(1-x) \\ \text{et } a^2x + b^2x = a^2 - ab + b^2 \\ \text{et } \underbrace{(a^2 + b^2)}_{\neq 0} x = a^2 - ab + b^2 \\ \text{et } x \in \left\{ \frac{a^2 - ab + b^2}{a^2 + b^2} \right\} \end{cases}$$

16)

$$(2a - 1)(x - b) = a(x + b) \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow (2a - 1) \cdot x - ax = (2a - 1) \cdot b + ab$$

$$\Leftrightarrow 2ax - x - ax = 2ab - b + ab$$

$$\Leftrightarrow (a - 1) \cdot x = 3ab - b$$

$$\Leftrightarrow (a - 1) \cdot x = b(3a - 1)$$

$$\Leftrightarrow \begin{cases} a = 1 & \text{et } 0 \cdot x = 2b \text{ et } \\ \text{ou} & \\ a \neq 1 & \text{et } x \in \left\{ \frac{b(3a - 1)}{a - 1} \right\} \end{cases} \begin{cases} b = 0 \text{ et } x \in \mathbb{R} \\ \text{ou} \\ b \neq 0 \text{ et } x \in \emptyset \end{cases}$$

$$(17) \quad \frac{2}{x^2+x} + \frac{m}{x-1} = \frac{m}{x} \quad \text{et } x \in \mathbb{R} - \underbrace{\{1; 0; -1\}}_D$$

$$\Leftrightarrow \frac{2}{x(x+1)} + \frac{m}{x-1} - \frac{m}{x} = 0$$

$$\Leftrightarrow \frac{2(x-1) + m(x)(x+1) - m(x^2-1)}{x(x+1)(x-1)} = 0$$

$$\Leftrightarrow \frac{2x-2 + \cancel{mx^2} + \cancel{mx} - \cancel{mx^2} + m}{x(x+1)(x-1)} = 0$$

$$\Leftrightarrow (m+2) \cdot x = (-m+2)$$

$$\Leftrightarrow \begin{cases} m = -2 & \text{et } 0 \cdot x = 4 \text{ et } x \in \emptyset \\ \text{ou} \\ m \neq -2 & \text{et } x = \frac{-m+2}{m+2} \text{ et } x \in \left\{ \frac{-m+2}{m+2} \right\} \end{cases}$$

$$\text{et } m=2 \text{ et } x=0 (\notin D) \text{ et } x \in \emptyset$$

$$\text{ou}$$

$$m=0 \text{ et } x=1 (\notin D) \text{ et } x \in \emptyset$$

$$\Leftrightarrow \begin{cases} m \in \{-2; 2; 0\} & \text{et } x \in \emptyset \\ \text{ou} \\ m \notin \{-2; 2; 0\} & \text{et } x \in \left\{ \frac{-m+2}{m+2} \right\} \end{cases}$$

coin brico du n°17

$$\frac{-m+2}{m+2} = \frac{1}{1}$$

$$\Leftrightarrow -m+2 = -m-2$$

$$\Leftrightarrow 0 \cdot m = -4$$

$$\Leftrightarrow m = \emptyset$$

$$\frac{-m+2}{m+2} = \frac{1}{1}$$

$$\Leftrightarrow -m+2 = m+2$$

$$\Leftrightarrow 0 = 2m$$

$$\Leftrightarrow m = 0$$

18) $\frac{a}{x+1} - \frac{b}{x-1} = \frac{a-b}{x^2-1}$ et $x \in \mathbb{R} - \underbrace{\{-1; 1\}}_D$

$$\Leftrightarrow \frac{a(x-1) - b(x+1)}{\cancel{x^2-1}} = \frac{a-b}{\cancel{x^2-1}}$$

$$\Leftrightarrow ax - a - bx - b = a - b$$

$$\Leftrightarrow (a-b) \cdot x = 2a$$

$$\Leftrightarrow \begin{cases} a=b \text{ et } 0 \cdot x = 2a \text{ et } \begin{cases} a=0 \text{ et } x \in \mathbb{R} - \{-1\} \\ \text{ou} \\ a \neq 0 \text{ et } x \in \emptyset \end{cases} \\ a \neq b \text{ et } x = \frac{2a}{a-b} \text{ et } x \in \left\{ \frac{2a}{a-b} \right\} \end{cases}$$

$$\text{et } \begin{cases} a = -b \text{ et } x = 1 (\notin D) \text{ et } x \in \emptyset \\ \text{ou} \\ b = 3a \text{ et } x = -1 (\notin D) \text{ et } x \in \emptyset \\ \text{ou} \\ a \neq -b \text{ et } b \neq 3a \text{ et } x \in \left\{ \frac{2a}{a-b} \right\} \end{cases}$$

$$\Leftrightarrow \begin{cases} (a,b) = (0,0) \\ a=b=0 \text{ et } x \in \mathbb{R} - \{-1; 1\} \\ \text{ou} \\ (a=b \text{ et } a \in \mathbb{R}^* \text{ ou } \begin{cases} a \neq 0 \text{ et } \\ a = -b \text{ ou } \begin{cases} a \neq 0 \text{ et } \\ b = 3a \end{cases} \end{cases} \\ (a=b \text{ ou } a = -b \text{ ou } b = 3a) \text{ et } a \neq b \\ \text{et } x \in \emptyset \\ \text{ou} \\ (a,b) \neq (0,0) \text{ et } \begin{cases} a \neq \pm b \\ \text{et} \\ b \neq 3a \end{cases} \text{ et } x \in \left\{ \frac{2a}{a-b} \right\} \\ \text{ou } a=0 \end{cases}$$

coin brico du n°18

$$\begin{array}{l|l} \frac{2a}{a-b} = 1 & \frac{2a}{a-b} = -1 \\ \Leftrightarrow 2a = a-b & \Leftrightarrow 2a = -a+b \\ \Leftrightarrow a = -b & \Leftrightarrow 3a = b \end{array}$$

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18 Trouver le domaine de définition et calculer les racines de la fonction rationnelle f .

$$1) f(x) = \frac{5x}{x^2 - 4}$$

$$2) f(x) = \frac{2x + 1}{(1 + 2x)^2}$$

$$3) f(x) = \frac{1}{x + 3} - \frac{6}{x}$$

$$4) f(x) = \frac{2}{4x - 1}$$

$$5) f(x) = \frac{1}{\frac{x}{x - 2}}$$

$$6) f(x) = \frac{1}{2x + 1} - \frac{3}{3x + 1}$$

$$7) f(x) = \frac{-x}{x + 5} + \frac{x}{7 + x}$$

$$8) f(x) = \frac{2 + x}{2 - x} + \frac{x + 3}{x - 3}$$

$$9) f(x) = \frac{2x - 1}{2x + 1} + \frac{1 - x}{x + 1}$$

$$10) f(x) = \frac{-2x}{x + 2} - \frac{3 - 2x^2}{x^2 - 4}$$

$$11) f(x) = \frac{1}{x} + \frac{1}{x + 1} - \frac{2}{x + 2}$$

$$12) f(x) = \frac{2x^2}{5 - x} - \frac{3x}{x - 5}$$

$$13) f(x) = \frac{x}{x + 1} - \frac{5 - x}{x - 3} - \frac{2x^2 - 1}{x^2 - 2x - 3}$$

$$14) f(x) = \frac{\frac{1}{x - 2}}{\frac{3}{x + 4}}$$

$$\textcircled{1)} \quad f(x) = \frac{5x}{x^2 - 4} = 0 \quad \text{et} \quad x \in \mathbb{R} - \{-2; 2\}$$

$$\Leftrightarrow 5x = 0$$

$$\Leftrightarrow x \in \{0\}$$

$$\textcircled{2) } f(x) = \frac{2x+1}{(1+2x)^2} = 0 \quad \text{et } x \in \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$\Leftrightarrow \frac{\cancel{2x+1}^1}{(2x+1)^2} = 0$$

$$\Leftrightarrow 1 = 0$$

$$\Leftrightarrow x \in \emptyset$$

$$\textcircled{6) \quad f(x) = \frac{1}{2x+1} - \frac{3}{3x+1} = 0 \quad \text{et } x \in \mathbb{R} - \left\{-\frac{1}{2}; -\frac{1}{3}\right\}$$

$$\Leftrightarrow \frac{1 \cdot (3x+1) - 3(2x+1)}{(2x+1)(3x+1)} = 0$$

$$\Leftrightarrow 3x+1-6x-3=0$$

$$\Leftrightarrow -3x = +2$$

$$\Leftrightarrow x = -\frac{2}{3}$$

$$\Leftrightarrow x \in \left\{-\frac{2}{3}\right\}$$

$$\textcircled{8) \quad f(x) = \frac{2+x}{2-x} + \frac{x+3}{x-3} = 0 \quad \text{et} \quad x \in \mathbb{R} - \{2; 3\}$$

$$\Leftrightarrow \frac{(x+2)(x-3) + (x+3)(2-x)}{(2-x)(x-3)} = 0$$

$$\Leftrightarrow \cancel{x^2} - x - 6 + (-\cancel{x^2} - x + 6) = 0$$

$$\begin{aligned} \Leftrightarrow \quad & -2x = 0 \\ & x = 0 \quad (\in D) \\ \Leftrightarrow \quad & x \in \{0\} \end{aligned}$$

$$\textcircled{11)} \quad f(x) = \frac{1}{x} + \frac{1}{x+1} - \frac{2}{x+2} = 0 \quad \text{et } x \in \mathbb{R} - \{0, -1, -2\}$$

$$\Leftrightarrow \frac{(x+1)(x+2) + x(x+2) - 2x(x+1)}{x(x+1)(x+2)} = 0$$

$$\Leftrightarrow \cancel{x^2} + 3x + 2 + \cancel{x^2} + 2x - \cancel{2x^2} - \cancel{2x} = 0$$

$$\Leftrightarrow 3x = -2 \quad \Leftrightarrow x = -\frac{2}{3} \quad (\in D)$$

$$\Leftrightarrow x \in \left\{-\frac{2}{3}\right\}$$

$$(12) \quad f(x) = \frac{2x^2}{5-x} - \frac{3x}{x-5} = 0 \quad \text{et } x \in \mathbb{R} - \{5\}$$

$$\Leftrightarrow \frac{-2x^2}{x-5} - \frac{3x}{x-5} = 0$$

$$\Leftrightarrow \frac{-2x^2 - 3x}{(x-5)} = 0$$

$$\Leftrightarrow +2x^2 + 3x = 0$$

$$\Leftrightarrow x(2x+3) = 0 \quad \Leftrightarrow x \in \left\{0; -\frac{3}{2}\right\}$$

$$13) \quad f(x) = \frac{x}{x+1} - \frac{5-x}{x-3} - \frac{2x^2-1}{x^2-2x-3} = 0 \quad \text{et } x \in \mathbb{R} \setminus \{-1; 3\}$$

$$\Leftrightarrow \frac{x(x-3) + (x-5)(x+1) - (2x^2-1)}{(x+1)(x-3)} = 0 \quad \left| \begin{array}{l} x^2-2x-3 = \\ (x+1)(x-3) \end{array} \right.$$

$$\Leftrightarrow \cancel{x^2} - 3x + \cancel{x^2} - 4x - 5 - \cancel{2x^2} + 1 = 0$$

$$\Leftrightarrow -7x = 4$$

$$\Leftrightarrow x \in \left\{ -\frac{4}{7} \right\}$$