SYSTEMES D'EQUATIONS

1 Méthodes de résolution

THEOREME 1

$$\begin{cases} a = b \\ \text{et} \\ c = d \end{cases} \Leftrightarrow \begin{cases} a + c = b + d \\ \text{et} \\ c = d \end{cases}$$

$$\begin{cases} a = b \\ \text{et} \\ c = d \end{cases} \Leftrightarrow \begin{cases} x \cdot a = x \cdot b \\ \text{et} \\ y \cdot c = y \cdot d \end{cases} \quad \text{et} \quad 0 \notin \{x,y\}$$

Exercice 6 *Résoudre dans* $\mathbb{R}^2$

1 $\begin{cases} 16x + 4y = 15 \\ \text{et} \\ 5x - 4y = 8 \end{cases}$

2 $\begin{cases} 2x + y = 1 \\ \text{et} \\ 2x + 5y = 3 \end{cases}$

3 $\begin{cases} x + 4y = 1 \\ \text{et} \\ 2x + y = 3 \end{cases}$

4 $\begin{cases} x + y = 3 \\ \text{et} \\ x - y = 2 \end{cases}$

5 $\begin{cases} 6x - 9y = 0 \\ \text{et} \\ 8x + 15y = 9 \end{cases}$

6 $\begin{cases} -4x + 5y = 2 \\ \text{et} \\ 12x + 10y = 56 \end{cases}$

Exercice 6 *Résoudre dans* $\mathbb{R}^2$

1 $\begin{cases} 16x + 4y = 15 \\ \text{et} \\ 5x - 4y = 8 \end{cases}$ et $(x,y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} y = \dfrac{15-16x}{4} \\ \text{et} \\ 5x - 4\left(\dfrac{15-16x}{4}\right) = 8 \end{cases}$

$\Leftrightarrow \begin{cases} y = \dfrac{15-16x}{4} = \dfrac{15-16\cdot\frac{23}{21}}{4} = \dfrac{-53}{84} \\ \text{et} \\ x = \dfrac{23}{21} \end{cases}$

$\Leftrightarrow (x,y) \in \left\{\left(\dfrac{23}{21} ; \dfrac{-53}{84}\right)\right\}$

$5x - 4\left(\dfrac{15-16x}{4}\right) = 8$

$\Leftrightarrow 5x - 15 + 16x = 8$

$\Leftrightarrow 21x = 23$

$\Leftrightarrow x = \dfrac{23}{21}$

$*$ $y = \dfrac{15-16\cdot\frac{23}{21}}{4}$

$= \dfrac{15\cdot 21 - 16\cdot 23}{4\cdot 21}$

$= \dfrac{-53}{84}$

3 $\begin{cases} x + 4y = 1 \\ \text{et} \\ 2x + y = 3 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$$\Leftrightarrow \begin{cases} x = 1 - 4y \\ 2(1-4y) + y = 3 \end{cases} \Leftrightarrow \begin{cases} x = 1 - 4y \\ -7y = 1 \end{cases} \Leftrightarrow \begin{cases} x = \frac{11}{7} \\ \text{et} \\ y = \frac{-1}{7} \end{cases}$$

$$\Leftrightarrow (x;y) \in \left\{ \left(\frac{11}{7} ; \frac{-1}{7} \right) \right\}$$

4 $\begin{cases} x + y = 3 & (1) \\ \text{et} \\ x - y = 2 & (2) \end{cases}$ et $(x, y) \in \mathbb{R}^2$

$\Longleftrightarrow \begin{cases} 2x = 5 & (3) = (1) + (2) \\ \text{et} \\ 2y = 1 & (4) = (1) - (2) \end{cases} \Longleftrightarrow \begin{cases} x = \frac{5}{2} \\ \text{et} \\ y = \frac{1}{2} \end{cases}$

$\Longleftrightarrow (x ; y) \in \left\{ \left(\frac{5}{2} ; \frac{1}{2} \right) \right\}$

5 $\begin{cases} 6x - 9y = 0 \\ \text{et} \\ 8x + 15y = 9 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} x = \dfrac{\overset{3}{\cancel{9}}y}{\underset{2}{\cancel{6}}} \\ \text{et} \\ x = \dfrac{9-15y}{8} \end{cases} \Leftrightarrow \begin{cases} x = \dfrac{3y}{2} \\ \text{et} \\ \dfrac{3y}{2} = \dfrac{9-15y}{8} \end{cases}$

$\dfrac{3y}{2} = \dfrac{9-15y}{8}$

$\Leftrightarrow 12y = 9 - 15y$

$\Leftrightarrow y = \dfrac{9}{27} = \dfrac{1}{3}$

$\Leftrightarrow \begin{cases} x = \dfrac{3}{2} \cdot \dfrac{1}{3} = \dfrac{1}{2} \\ \text{et} \\ y = \dfrac{1}{3} \end{cases}$

$\Leftrightarrow (x;y) \in \left\{\left(\dfrac{1}{2};\dfrac{1}{3}\right)\right\}$

6 $\begin{cases} -4x + 5y = 2 \\ \text{et} \\ 12x + 10y = 56 \end{cases}$ et $(x,y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} -4x + 5y = 2 & (1) \\ 6x + 5y = 28 & (2) \end{cases}$ $\Leftrightarrow \begin{cases} -10x = -26 & (3) = (1) - (2) \\ \text{et} \\ 25y = 62 & (4) = 3 \cdot (1) + 2 \cdot (2) \end{cases}$

$\Leftrightarrow \begin{cases} x = \frac{26}{10} = \frac{13}{5} \\ y = \frac{62}{25} \end{cases}$ $\Leftrightarrow (x;y) \in \left\{ \left(\frac{13}{5} ; \frac{62}{25} \right) \right\}$

Exercice 7 *Résoudre dans* $\mathbb{R}^2$ *avec l'une des trois méthodes*

1 $\begin{cases} 2x + 4y = 42 \\ -5x + 7y = 31 \end{cases}$

2 $\begin{cases} 3x - 8y = 36 \\ 4x - 5y = -14 \end{cases}$

3 $\begin{cases} 6x + 5y = 8 \\ 3x + \dfrac{y}{2} = -4 \end{cases}$

4 $\begin{cases} 5x - \dfrac{y}{4} = 33 \\ -10x + \dfrac{y}{2} = -68 \end{cases}$

5 $\begin{cases} mx + 5y = 10 \\ 3x - 2y = -2 \end{cases}$

6 $\begin{cases} x + 2y = 17 \\ -2x + 3y = 1 \end{cases}$

7 $\begin{cases} 2x = 3y \\ x + y = \dfrac{5}{12} \end{cases}$

8 $\begin{cases} x^2 + 2y^2 = 22 \\ 2x^2 - y^2 = -1 \end{cases}$

9 $\begin{cases} \dfrac{x}{3} - \dfrac{y}{2} = -2 \\ -x + \dfrac{3y}{2} = 6 \end{cases}$

10 $\begin{cases} \dfrac{x}{2} = 4y \\ 3 = 2y - x \end{cases}$

11 $\begin{cases} 11x + 18y = 1 \\ 22x + 36y = 3 \end{cases}$

12 $\begin{cases} 2x = -\dfrac{1}{3} \\ -7y = 1 + x \end{cases}$

1 $\begin{cases} 2x + 4y = 42 \\ -5x + 7y = 31 \end{cases}$ et $(x,y) \in \mathbb{R}^2$

$$\Leftrightarrow \begin{cases} x + 2y = 21 \\ -5 \cdot x + 7 \cdot y = 31 \end{cases} \Leftrightarrow \begin{cases} x = 21 - 2y \\ x = \dfrac{31 - 7y}{-5} \end{cases}$$

$$\Leftrightarrow \begin{cases} 21 - 2y = \dfrac{31 - 7y}{-5} \\ x = \cancel{\dfrac{31 - 7y}{-5}}\ 21 - 2y \end{cases} \ldots \Leftrightarrow \begin{cases} y = 8 \\ x = 5 \end{cases}$$

$$\Leftrightarrow (x ; y) \in \{ (5 ; 8) \}$$

2 $\begin{cases} 3x - 8y = 36 \\ 4x - 5y = -14 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$$\iff \left\{\begin{array}{l|l|l} 3x - 8y = 36 & 4 & 5 \\ 4x - 5y = -14 & -3 & -8 \end{array}\right. \iff \begin{cases} 0 \cdot x - 17y = 186 \\ -17 \cdot x + 0 \cdot y = 292 \end{cases}$$

$$\iff \begin{cases} y = \dfrac{-186}{17} \\ x = \dfrac{-292}{17} \end{cases} \iff (x;y) \in \left\{ \left(\dfrac{-292}{17} ; \dfrac{-186}{17} \right) \right\}$$

(5) $\begin{cases} mx + 5y = 10 \\ 3x - 2y = -2 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$\Longleftrightarrow \begin{cases} y = \dfrac{10 - mx}{5} \\ y = \dfrac{+2 + 3x}{+2} \end{cases} \Longleftrightarrow \begin{cases} \dfrac{10 - mx}{5} = \dfrac{3x+2}{2} \\ y = \dfrac{3x+2}{2} \end{cases}$

$\Longleftrightarrow \begin{cases} m = \dfrac{-15}{2} \text{ et } (x;y) \in \emptyset \\ \text{ou} \\ m \neq \dfrac{-15}{2} \text{ et} \\ (x;y) \in \left\{ \left(\dfrac{10}{2m+15} ; \dfrac{2m+30}{2m+15} \right) \right\} \end{cases}$

$\dfrac{10 - mx}{5} = \dfrac{3x+2}{2}$

$\Longleftrightarrow 20 - 2mx = 15x + 10$

$\Longleftrightarrow 10 = (2m+15)x$

$\Longleftrightarrow \begin{cases} m = \dfrac{-15}{2} \text{ et } 0 \cdot x = 10 \text{ et } x \in \emptyset \\ \text{ou} \\ m \neq \dfrac{-15}{2} \text{ et } x = \dfrac{10}{2m+15} \end{cases}$

$y = \dfrac{2 + 3\left(\dfrac{10}{2m+15}\right)}{2}$

$= \dfrac{4m + 30 + 30}{2(2m+15)} = \dfrac{2m+30}{2m+15}$

(8) $\begin{cases} x^2 + 2y^2 = 22 \\ 2x^2 - y^2 = -1 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$$\Leftrightarrow u = x^2 \text{ et } t = y^2 \text{ et } \begin{cases} u + 2t = 22 \\ 2u - t = -1 \end{cases} \begin{array}{|c|c} -2 & 1 \\ 1 & 2 \end{array} \Leftrightarrow \begin{cases} u = x^2 \text{ et } t = y^2 \\ \text{et } \; 0\cdot u + -5t = -45 \\ \quad\; 5u + 0\cdot t = 20 \end{cases}$$

$$\Leftrightarrow \begin{cases} u = x^2 = 4 \\ \text{et} \\ t = y^2 = 9 \end{cases} \Leftrightarrow \begin{cases} x \in \{-2;2\} \\ \text{et} \\ y \in \{-3;3\} \end{cases}$$

$$\Leftrightarrow (x;y) \in \{(-2;-3);(-2;3);(2;-3);(2;3)\}$$

9 $\begin{cases} \dfrac{x}{3} - \dfrac{y}{2} = -2 \\ -x + \dfrac{3y}{2} = 6 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$$\Longleftrightarrow \begin{cases} 2x - 3y = -12 \\ -2x + 3y = 12 \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

$$\Longleftrightarrow \begin{cases} 0 \cdot x + 0 \cdot y = 0 \\ 2x - 3y = -12 \end{cases}$$

$$\Longleftrightarrow \begin{cases} (x;y) \in \mathbb{R}^2 \\ 2x - 3y = -12 \end{cases}$$

$$\left(\Longleftrightarrow (x,y) \in \left\{ (x,y) \in \mathbb{R}^2 \mid 2x - 3y = -12 \right\}\right)$$

$$\Longleftrightarrow (x;y) \in \left\{ \left(k ; \frac{12+2k}{3}\right) \middle| k \in \mathbb{R} \right\}$$

$$\Longleftrightarrow (x;y) \in \left\{ \left(\frac{-12+3k}{2}, k\right) \middle| k \in \mathbb{R} \right\}$$

(11) $\begin{cases} 11x + 18y = 1 \\ 22x + 36y = 3 \end{cases} \begin{array}{|l} 2 \\ -1 \end{array}$ et $(x;y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} 0 \cdot x + 0 \cdot y = -1 \\ 11x + 18y = 1 \end{cases} \Leftrightarrow (x;y) \in \emptyset$