

SYSTEMES D'EQUATIONS

1 Méthodes de résolution

$$13 \quad \begin{cases} \frac{5(x+y)}{3} = 15 \\ x - 2y = -3 \end{cases}$$

$$14 \quad \begin{cases} 2x + y + 7 = -7 - 3y \\ 4x + 4y + 4 = x - 7 \end{cases}$$

$$15 \quad \begin{cases} \frac{x+y}{2} = \frac{x-y}{3} \\ x + 4y = -\frac{1}{2} \end{cases}$$

$$16 \quad \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{7}{12} \\ \frac{1}{x} - \frac{1}{y} = -\frac{1}{12} \end{cases}$$

$$17 \quad \begin{cases} \frac{1}{x+1} + \frac{1}{y-2} = \frac{7}{12} \\ \frac{1}{x+1} - \frac{1}{y-2} = -\frac{1}{2} \end{cases}$$

$$18 \quad \begin{cases} x + \frac{8}{y-1} = -3 \\ -2x + \frac{12}{y-1} = -3 \end{cases}$$

$$19 \quad \begin{cases} \frac{7}{x} + \frac{4}{y} = \frac{1}{2} \\ \frac{3}{x} - \frac{5}{y} = \frac{3}{14} \end{cases}$$

$$20 \quad \begin{cases} \frac{x+y}{xy} = \frac{3}{4} \\ \frac{x-y}{xy} = \frac{1}{4} \end{cases}$$

$$21 \quad \begin{cases} 2(x+2y) = 0 \\ -3(-y+3x) = 0 \end{cases}$$

$$22 \quad \begin{cases} \frac{x}{3} = 2y - 1 \\ 3 = 2y - x \end{cases}$$

$$23 \quad \begin{cases} \frac{x}{3} - 5y + 8 = \frac{x}{2} - 3 \\ \frac{y}{2} - \frac{x}{3} + 4 = y + 1 \end{cases}$$

$$24 \quad \begin{cases} 2y + 3x - \frac{43}{12} = 0 \\ -5x + 3y = -\frac{7}{4} \end{cases}$$

$$25 \quad \begin{cases} 2x + (m-1)y = 1 \\ 3x + y = 0 \end{cases}$$

$$26 \quad \begin{cases} mx + y = 2 \\ 2x - y = 1 \end{cases}$$

$$27 \quad \begin{cases} 5x - 2y = m \\ 2x + 3y = 17 \end{cases}$$

$$28 \quad \begin{cases} (m+2)x + y = 1 \\ 3x + 2y = 13 \end{cases}$$

$$29 \quad \begin{cases} x + my = 2 \\ mx + 3y = 3 \end{cases}$$

$$30 \quad \begin{cases} x + 4y = m+3 \\ 7x - y = 1 \end{cases}$$

$$31 \quad \begin{cases} x + y = 3 \\ mx - 2y = 5 \end{cases}$$

$$\textcircled{15} \begin{cases} \frac{x+y}{2} = \frac{x-y}{3} \\ x + 4y = -\frac{1}{2} \end{cases} \text{ et } (x; y) \in \mathbb{R}$$

$$\Leftrightarrow \begin{cases} 3x + 3y = 2x - 2y \\ 2x + 8y = -1 \end{cases} \Leftrightarrow \begin{cases} x + 5y = 0 \\ 2x + 8y = -1 \end{cases}$$

$$\Leftrightarrow D = \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix} = -2 \text{ et } D_x = \begin{vmatrix} 0 & 5 \\ -1 & 8 \end{vmatrix} = 5 \text{ et } D_y = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1$$

$$\text{et } (x; y) \in \left\{ \left(-\frac{5}{2} ; \frac{1}{2} \right) \right\}$$

exercice 7 $m=17$

$$\begin{cases} \frac{1}{x+1} + \frac{1}{y-2} = \frac{7}{12} \\ \frac{1}{x+1} - \frac{1}{y-2} = \frac{-1}{2} \end{cases} \text{ et } (x,y) \in \underset{\substack{\downarrow \\ x \neq -1}}{\mathbb{R} - \{-1\}} \times \underset{\substack{\downarrow \\ y \neq 2}}{\mathbb{R} - \{2\}}$$

$$\Leftrightarrow u = \frac{1}{x+1} \text{ et } t = \frac{1}{y-2} \text{ et } \begin{cases} u + t = \frac{7}{12} & (1) \\ u - t = \frac{-1}{2} & (2) \end{cases}$$

$$\Leftrightarrow u = \frac{1}{x+1} \text{ et } t = \frac{1}{y-2} \text{ et } \begin{cases} 2u = \frac{7}{12} - \frac{1}{2} \text{ et } u = \frac{1}{24} & (\text{avec } (1) + (2)) \\ 2t = \frac{7}{12} + \frac{1}{2} \text{ et } t = \frac{13}{24} & (\text{avec } (1) - (2)) \end{cases}$$

$$\Leftrightarrow \frac{1}{x+1} = \frac{1}{24} \text{ et } x = 23$$

$$\frac{1}{y-2} = \frac{13}{24} \text{ et } y-2 = \frac{24}{13} \text{ et } y = 2 + \frac{24}{13} = \frac{50}{13}$$

$$\Rightarrow (x,y) \in \left\{ \left(23 ; \frac{50}{13} \right) \right\}$$

$$18 \quad \begin{cases} x + \frac{8}{y-1} = -3 \\ -2x + \frac{12}{y-1} = -3 \end{cases} \quad \text{et } (x; y) \in \mathbb{R} \times \mathbb{R} - \{1\}$$

$$\Leftrightarrow \begin{cases} t = \frac{1}{y-1} \quad \text{et} \\ x + 8t = -3 \\ -2x + 12t = -3 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \begin{vmatrix} 3 \\ -2 \end{vmatrix} \Leftrightarrow \begin{cases} t = \frac{1}{y-1} \quad \text{et} \\ 0 \cdot x + 28t = -9 \\ 7 \cdot x + 0 \cdot t = -3 \end{cases}$$

$$\Leftrightarrow \begin{cases} t = \frac{1}{y-1} = \frac{-9}{28} \\ x = \frac{-3}{7} \end{cases} \Leftrightarrow \begin{cases} y = \frac{-19}{9} \\ x = \frac{-3}{7} \end{cases}$$

$$\Leftrightarrow (x; y) \in \left\{ \left(\frac{-3}{7} ; \frac{-19}{9} \right) \right\}$$

$$\begin{aligned} & \frac{1}{y-1} = \frac{-9}{28} \\ \Leftrightarrow 28 &= -9y + 9 \\ \Leftrightarrow y &= \frac{-19}{9} \end{aligned}$$

COMPLÉMENT

(18) Sans Cramer :

$$\begin{cases} \frac{x+y}{2} = \frac{x-y}{3} \\ x+4y = -\frac{1}{2} \end{cases} \quad \text{et } (x,y) \in \mathbb{R}^2$$

$$\Leftrightarrow \dots \begin{cases} x+5y=0 \\ 2x+8y=-1 \end{cases} \Leftrightarrow \begin{cases} 2x = -10y \\ 2x = -1-8y \end{cases} \Leftrightarrow \begin{cases} -10y = -1-8y \\ x = -5y \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \frac{1}{2} \\ x = -\frac{5}{2} \end{cases} \Leftrightarrow (x,y) \in \left\{ \left(-\frac{5}{2}, \frac{1}{2} \right) \right\}$$

(19) $\begin{cases} \frac{7}{x} + \frac{4}{y} = \frac{1}{2} \\ \frac{3}{x} - \frac{5}{y} = \frac{3}{14} \end{cases} \quad \text{et } (x,y) \in \mathbb{R}^* \times \mathbb{R}^*$

$$\Leftrightarrow \dots \begin{cases} 7t + 4u = \frac{1}{2} \\ 3t - 5u = \frac{3}{14} \end{cases} \begin{vmatrix} 3 & 5 \\ -7 & 4 \end{vmatrix} \Leftrightarrow \begin{cases} 0 \cdot t + 47u = 0 \\ 47 \cdot t + 0 \cdot u = \frac{47}{14} \end{cases}$$

$$\Leftrightarrow \begin{cases} u = 0 = \frac{1}{y} \\ t = \frac{1}{14} = \frac{1}{x} \end{cases} \Leftrightarrow \begin{cases} y \in \emptyset \\ x = 14 \end{cases} \Leftrightarrow (x,y) \in \emptyset$$

$$\textcircled{19} \quad \begin{cases} \frac{7}{x} + \frac{4}{y} = \frac{1}{2} \\ \frac{3}{x} - \frac{5}{y} = \frac{3}{14} \end{cases} \quad \text{et } (x; y) \in \mathbb{R}^* \times \mathbb{R}^*$$

$$\Leftrightarrow \begin{cases} 7t + 4u = \frac{1}{2} \\ 3t - 5u = \frac{3}{14} \end{cases} \quad \text{et} \quad \begin{cases} t = \frac{1}{x} \\ u = \frac{1}{y} \end{cases}$$

$$\Leftrightarrow \begin{cases} D = \begin{vmatrix} 7 & 4 \\ 3 & -5 \end{vmatrix} = -47 \neq 0 \\ D_t = \begin{vmatrix} \frac{1}{2} & 4 \\ \frac{3}{14} & -5 \end{vmatrix} = \frac{5}{2} - \frac{6}{7} = \frac{-47}{14} \\ D_u = \begin{vmatrix} 7 & \frac{1}{2} \\ 3 & \frac{3}{14} \end{vmatrix} = \frac{3}{2} - \frac{3}{2} = 0 \end{cases}$$

$$\text{et } \frac{t}{x} = \frac{\frac{-47}{14}}{-47} = \frac{1}{14}$$

$$\frac{u}{y} = 0$$

$$\Leftrightarrow \begin{cases} t = \frac{1}{14} = \frac{1}{x} \\ u = 0 = \frac{1}{y} \end{cases} \quad \Leftrightarrow (x; y) \in \emptyset$$

$$20 \quad \begin{cases} \frac{x+y}{xy} = \frac{3}{4} \\ \frac{x-y}{xy} = \frac{1}{4} \end{cases} \quad \text{et } (x; y) \in \mathbb{R}^* \times \mathbb{R}^*$$

$$\Leftrightarrow \begin{cases} \frac{\cancel{x}}{\cancel{x}y} + \frac{\cancel{y}}{x\cancel{y}} = \frac{3}{4} \\ \frac{1}{y} - \frac{1}{x} = \frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} t = \frac{1}{y} & \text{et } u = \frac{1}{x} \\ t + u = \frac{3}{4} \\ t - u = \frac{1}{4} \end{cases} \left| \begin{array}{c|c} 1 & 1 \\ 1 & -1 \end{array} \right|$$

$$\Leftrightarrow \begin{cases} t = \frac{1}{y} & \text{et } u = \frac{1}{x} \\ 2t = 1 & \text{et } t = \frac{1}{2} \\ 2u = \frac{1}{2} & \text{et } u = \frac{1}{4} \end{cases} \Leftrightarrow \begin{cases} t = \frac{1}{y} = \frac{1}{2} & \text{et } y = 2 \\ u = \frac{1}{x} = \frac{1}{4} & \text{et } x = 4 \end{cases}$$

$$\Leftrightarrow (x; y) \in \{(4; 2)\}$$

$$29 \quad \begin{cases} x + my = 2 \\ mx + 3y = 3 \end{cases} \quad \text{et } (x; y) \in \mathbb{R}^2$$

$$\Leftrightarrow \begin{cases} x = 2 - my \\ mx = 3 - 3y \end{cases} \Leftrightarrow \begin{cases} m=0 \text{ et } \begin{cases} x = 2 \\ 0 \cdot x = 3 - 3y \end{cases} \\ \text{ou} \\ m \neq 0 \text{ et } \begin{cases} x = 2 - my \\ x = \frac{3-3y}{m} \end{cases} \end{cases}$$

$$\Leftrightarrow \begin{cases} m=0 \text{ et } (x; y) \in \{(2; 1)\} \\ \text{ou} \\ m \neq 0 \text{ et } \begin{cases} 2 - my = \frac{3-3y}{m} \\ \text{et} \\ x = 2 - my \end{cases} \end{cases} \text{ et } \begin{cases} m \in \{-\sqrt{3}; \sqrt{3}\} \text{ et } y \in \emptyset \\ \text{ou} \\ m \notin \{-\sqrt{3}; \sqrt{3}\} \text{ et} \\ y = \frac{2m-3}{m^2-3} \\ \text{et } x = 2 - my \end{cases}$$

$$\Leftrightarrow \begin{cases} m=0 \text{ et } (x; y) \in \{(2; 1)\} \\ \text{ou} \\ m \in \{-\sqrt{3}; \sqrt{3}\} \text{ et } (x; y) \in \emptyset \\ \text{ou} \\ m \notin \{-\sqrt{3}; +\sqrt{3}, 0\} \text{ et } (x; y) \in \left\{ \left(\frac{3m-6}{m^2-3} ; \frac{2m-3}{m^2-3} \right) \right\} \end{cases}$$

$$\text{c.l. : } x = 2 - m \cdot \left(\frac{2m-3}{m^2-3} \right) = \frac{2m^2 - 6 - 2m^2 + 3m}{m^2-3}$$

$$= \frac{3m-6}{m^2-3}$$

Carmin l'icor :

$$2-my = \frac{3-3y}{m} \Leftrightarrow 2m-m^2y = 3-3y$$

$$\Leftrightarrow 2m-3 = (m^2-3)y$$

$$\Leftrightarrow \begin{cases} m \in \{\sqrt{3}; -\sqrt{3}\} & \text{et } 0.y = \pm 2\sqrt{3}-3 (\neq 0) \\ \text{ou} \\ m \notin \{\sqrt{3}; -\sqrt{3}\} & \text{et } y = \frac{2m-3}{m^2-3} \end{cases}$$