SYSTEMES D'EQUATIONS

2 Méthode des déterminants

2.1 Théorème de Cramer

exercice 14 :

1 $\begin{cases} (m-4)x + my = -2 \\ 3x + y = 3 \end{cases}$ 2 $\begin{cases} mx + 3y = 5 \\ 6x + 2y = 3 \end{cases}$ 3 $\begin{cases} 7x - (m+5)y = 0 \\ 2x + y = 1 \end{cases}$

4 $\begin{cases} 4x + my = 3 \\ mx + 4y = m+1 \end{cases}$ 5 $\begin{cases} ax + by = ab+1 \\ abx + ay = a^2+b \end{cases}$ 6 $\begin{cases} x - (m+1)y = m \\ (m+2)x + (m+1)y = -1 \end{cases}$

7 $\begin{cases} (a+b)x + by = a \\ (a+b)x + ay = b \end{cases}$ 8 $\begin{cases} y = mx + 2m \\ 2x = y - 3m \end{cases}$ 9 $\begin{cases} 2x = my + m \\ 3x + 2y = 1 \end{cases}$

2 $\begin{cases} mx + 3y = 5 \\ 6x + 2y = 3 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$\Leftarrow$ $D = \begin{vmatrix} m & 3 \\ 6 & 2 \end{vmatrix} = 2m - 18 = 2(m-9)$

$D_x = \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} = 1$ et $D_y = \begin{vmatrix} m & 5 \\ 6 & 3 \end{vmatrix} = 3m - 30 = 3(m-10)$

et $\begin{cases} m \neq 9 \text{ et } D \neq 0 \text{ et } (x;y) \in \left\{ \left(\dfrac{1}{2(m-9)} ; \dfrac{3(m-10)}{2(m-9)} \right) \right\} \\ \text{ou} \\ m = 9 \text{ et } D = 0 \text{ et } D_x = 1 \neq 0 \text{ et } (x;y) \in \emptyset \end{cases}$

3 $\begin{cases} 7x - (m+5)y = 0 \\ 2x + y = 1 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$$\Leftrightarrow \begin{cases} D = \begin{vmatrix} 7 & -m-5 \\ 2 & 1 \end{vmatrix} = 7 - 2(-m-5) = 2m + 17 \\ D_x = \begin{vmatrix} 0 & -m-5 \\ 1 & 1 \end{vmatrix} = m+5 \\ D_y = \begin{vmatrix} 7 & 0 \\ 2 & 1 \end{vmatrix} = 7 \end{cases}$$

et $m = \frac{-17}{2}$ et $D = 0$ et $D_y \neq 0$ $\left(\text{et } D_x = \frac{-7}{2} \neq 0\right)$

et $(x;y) \in \emptyset$

ou

$m \neq \frac{-17}{2}$ et $D \neq 0$ et $(x;y) \in \left\{\left(\frac{m+5}{2m+17} ; \frac{7}{2m+17}\right)\right\}$

(4) $\begin{cases} 4x + my = 3 \\ mx + 4y = m+1 \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} D = \begin{vmatrix} 4 & m \\ m & 4 \end{vmatrix} = 16 - m^2 = (4-m)(4+m) \\ D_x = \begin{vmatrix} 3 & m \\ m+1 & 4 \end{vmatrix} = 12 - m(m+1) = -m^2 - m + 12 \\ \quad = -(m^2+m-12) = -(m+4)(m-3) \\ D_y = \begin{vmatrix} 4 & 3 \\ m & m+1 \end{vmatrix} = 4m+4-3m = m+4 \end{cases}$

et $m = 4$ et $D=0$ et $D_x = -8 \neq 0$ $(\text{et } D_y = 8 \neq 0)$

et $(x;y) \in \emptyset$

ou

$m = -4$ et $D = 0 = D_x = D_y$ (syst. ind.)

et $\begin{cases} 4x - 4y = 3 \\ \cancel{-4x + 4y = -3} \end{cases}$

et $(x;y) \in \left\{ \left(k \,;\, \frac{4k-3}{4}\right) \middle| \, k \in \mathbb{R} \right\}$

ou

$m \notin \{4;-4\}$ et $(x;y) \in \left\{ \left(\frac{-\cancel{(m+4)}(m-3)}{(4-m)\cancel{(4+m)}} \,;\, \frac{\overset{1}{\cancel{m+4}}}{(4-m)\cancel{(4+m)}} \right) \right\}$

et $(x;y) \in \left\{ \left(\frac{m-3}{m-4} \,;\, \frac{1}{4-m} \right) \right\}$