SYSTEMES D'EQUATIONS

2 Méthode des déterminants

2.1 Théorème de Cramer

exercice 14 :

1 $\begin{cases} (m-4)x + my = -2 \\ 3x + y = 3 \end{cases}$ 2 $\begin{cases} mx + 3y = 5 \\ 6x + 2y = 3 \end{cases}$ 3 $\begin{cases} 7x - (m+5)y = 0 \\ 2x + y = 1 \end{cases}$

4 $\begin{cases} 4x + my = 3 \\ mx + 4y = m+1 \end{cases}$ 5 $\begin{cases} ax + by = ab+1 \\ abx + ay = a^2+b \end{cases}$ 6 $\begin{cases} x - (m+1)y = m \\ (m+2)x + (m+1)y = -1 \end{cases}$

7 $\begin{cases} (a+b)x + by = a \\ (a+b)x + ay = b \end{cases}$ 8 $\begin{cases} y = mx + 2m \\ 2x = y - 3m \end{cases}$ 9 $\begin{cases} 2x = my + m \\ 3x + 2y = 1 \end{cases}$

5 $\begin{cases} ax + by = ab + 1 \\ abx + ay = a^2 + b \end{cases}$ $\quad$ et $(x,y) \in \mathbb{R}^2$

$$\Leftarrow \begin{cases} D = \begin{vmatrix} a & b \\ ab & a \end{vmatrix} = a^2 - ab^2 = a(a-b^2) \\ D_x = \begin{vmatrix} ab+1 & b \\ a^2+b & a \end{vmatrix} = \cancel{a^2b} + a - \cancel{a^2b} - b^2 = a - b^2 \\ D_y = \begin{vmatrix} a & ab+1 \\ ab & a^2+b \end{vmatrix} = a^3 + \cancel{ab} - a^2b^2 - \cancel{ab} = a^2(a-b^2) \end{cases}$$

et $a = b^2$ et $D = 0 = D_x = D_y$ et

$$\begin{cases} b^2x + by = b^3 + 1 \\ b^3x + b^2y = b^4 + b \end{cases}$$

et $b = 0$ et $\begin{cases} 0\cdot x + 0y = 1 \\ 0\cdot x + 0\cdot y = 0 \end{cases}$

et $(x;y) \in \emptyset$

ou $b \neq 0$

$\begin{cases} b^2x + by = b^3 + 1 \\ \cancel{b^2x + by = b^3 + 1} \end{cases}$ et $(x;y) \in \left\{ \left(k ; \dfrac{b^3+1-kb^2}{b}\right) \middle| k \in \mathbb{R} \right\}$

ou

$a = 0$ (et $b \neq 0$) et $D = 0$ et $D_x = -b^2 \neq 0$ et $D_y = 0$

et $\begin{cases} 0x + by = 1 \\ \text{et} \\ 0\cdot x + 0\cdot y = b \ (\neq 0) \end{cases}$ et $(x;y) \in \emptyset$

ou

$a \neq 0$ et $a \neq b^2$ et $D \neq 0$

et $(x;y) \in \left\{ \left(\dfrac{1}{a} ; a \right) \right\}$

$$6 \begin{cases} x - (m+1)y = m \\ (m+2)x + (m+1)y = -1 \end{cases} \quad \text{et } (x, y) \in \mathbb{R}^2$$

$$\Leftarrow, \quad D = \begin{vmatrix} 1 & -(m+1) \\ (m+2) & (m+1) \end{vmatrix} = (m+1) + (m+1)(m+2) = (m+1)(m+3)$$

et

$$D_x = \begin{vmatrix} m & -(m+1) \\ (-1) & (m+1) \end{vmatrix} = m(m+1) - (m+1) = (m+1)(m-1)$$

et

$$D_y = \begin{vmatrix} 1 & m \\ (m+2) & -1 \end{vmatrix} = -1 - m(m+2) = -m^2 - 2m - 1$$

$$= -(m^2 + 2m + 1)$$

$$= -(m+1)^2$$

et $\begin{cases} m = -1 \text{ et } D = 0 = D_x = D_y \\ \quad \text{et } \begin{cases} x + 0 \cdot y = -1 \\ \cancel{x + 0 \cdot y = -1} \end{cases} \text{ et } (x;y) \in \{(-1\ ;\ k) \mid k \in \mathbb{R}\} \\ \text{ou} \\ m = -3 \text{ et } D = 0 \text{ et } D_y = -4 \neq 0 \text{ et } (x;y) \in \emptyset \\ \text{ou} \\ m \notin \{-1 ; -3\} \text{ et } D \neq 0 \text{ et } (x;y) \in \left\{\left(\frac{m-1}{m+3} ; \frac{-(m+1)}{m+3}\right)\right\} \end{cases}$

7 $\begin{cases} (a+b)x + by = a \\ (a+b)x + ay = b \end{cases}$ et $(x;y) \in \mathbb{R}^2$

$\Leftrightarrow \begin{cases} D = \begin{vmatrix} a+b & b \\ a+b & a \end{vmatrix} = (a+b)a - (a+b)b = (a+b)(a-b) \\ \text{et} \\ D_x = \begin{vmatrix} a & b \\ b & a \end{vmatrix} = a^2 - b^2 = (a+b)(a-b) \\ \text{et} \\ D_y = \begin{vmatrix} a+b & a \\ a+b & b \end{vmatrix} = (a+b)(b-a) \end{cases}$

et $a = -b$ et $D = 0 = D_x = D_y$

et $\begin{cases} 0x + by = -b \\ \cancel{0.x - by = b} \end{cases}$ et

$\begin{cases} b = 0 \text{ et } 0.x + 0.y = 0 \text{ et } (x;y) \in \mathbb{R}^2 \\ \text{et} \\ b \neq 0 \text{ et } y = -1 \text{ et } (x;y) \in \{(k;-1) \mid k \in \mathbb{R}\} \end{cases}$

ou

$a = b$ et $D = 0 = D_x = D_y$

et $a \neq 0$ et $\begin{cases} 2ax + ay = a \\ \cancel{2ax + ay = a} \end{cases} \xrightarrow{:a}$ et $2x + y = 1$

et $(x;y) \in \{(k;1-2k) \mid k \in \mathbb{R}\}$

ou

$a \neq b$ et $a \neq -b$ et $D \neq 0$

et $(x;y) \in \{(1;-1)\}$

8 $\begin{cases} y = mx + 2m \\ 2x = y - 3m \end{cases}$

$\Leftrightarrow \begin{cases} mx - y = -2m \\ 2x - y = -3m \end{cases}$

$\Leftrightarrow \begin{cases} D = \begin{vmatrix} m & -1 \\ 2 & -1 \end{vmatrix} = -m+2 \\ D_x = \begin{vmatrix} -2m & -1 \\ -3m & -1 \end{vmatrix} = 2m - 3m = -m \\ D_y = \begin{vmatrix} m & -2m \\ 2 & -3m \end{vmatrix} = -3m^2 + 4m \\ \qquad = m(-3m+4) \end{cases}$ et $(x;y) \in \mathbb{R}^2$

et $m = 2$ et $D = 0$ et $D_x = -2 \neq 0$ et $(x;y) \in \emptyset$
$(\text{et } D_y = -4 \neq 0)$

ou

$m \neq 2$ et $D \neq 0$ et $(x;y) \in \left\{ \left(\frac{-m}{-m+2} ; \frac{-3m^2+4m}{-m+2} \right) \right\}$

$\in \left\{ \left(\frac{m}{m-2} ; \frac{3m^2-4m}{m-2} \right) \right\}$