

Exercise 11

$$1 \quad \begin{cases} 2x + (m-1)y = 1 \\ 4x + 2y = 7 \end{cases}$$

$$2 \quad \begin{cases} 2mx + y = 3 \\ \frac{x}{3} - y = 1 \end{cases}$$

$$3 \quad \begin{cases} 4x + 2y = m \\ 3x - y = 1 \end{cases}$$

$$4 \quad \begin{cases} (m+2)x + y = 2 \\ (m+2)y + x = 4 \end{cases}$$

$$5 \quad \begin{cases} (m^2+1)x + y = 0 \\ 2x + 5y = 1 \end{cases}$$

$$6 \quad \begin{cases} x + my = 0 \\ 2x - 3y = 0 \end{cases}$$

$$1) \begin{cases} 2x + (m-1)y = 1 \\ 4x + 2y = 7 \end{cases} \text{ et } (x, y) \in \mathbb{R} \times \mathbb{R}$$

$$\Leftrightarrow D = \begin{vmatrix} 2 & m-1 \\ 4 & 2 \end{vmatrix} = 4 - 4m + 4 = -4m + 8 = (-4)(m-2)$$

$$D_x = \begin{vmatrix} 1 & m-1 \\ 7 & 2 \end{vmatrix} = 2 - 7m + 7 = -7m + 9$$

$$D_y = \begin{vmatrix} 2 & 1 \\ 4 & 7 \end{vmatrix} = 14 - 4 = 10$$

et $m=2$ et $D=0$ et $D_x = -6$ et $D_y = 10$ et $(x, y) \in \emptyset$
(système impossible)

ou
 $m \neq 2$ et $x = \frac{D_x}{D} = \frac{-7m+9}{-4(m-2)} = \frac{7m-9}{4(m-2)}$

et $y = \frac{D_y}{D} = \frac{10}{-4(m-2)}$

et $(x, y) \in \left\{ \left(\frac{7m-9}{4(m-2)} ; \frac{-5}{2(m-2)} \right) \right\}$

$$2) \begin{cases} 2mx + y = 3 \\ \frac{x}{3} - y = 1 \end{cases} \text{ et } (x, y) \in \mathbb{R} \times \mathbb{R}$$

$$\Leftrightarrow \begin{cases} 2mx + y = 3 \\ x - 3y = 3 \end{cases} \Leftrightarrow \begin{cases} D = \begin{vmatrix} 2m & 1 \\ 1 & -3 \end{vmatrix} = -6m - 1 \\ D_x = \begin{vmatrix} 3 & 1 \\ 3 & -3 \end{vmatrix} = -10 - 9 - 3 = -12 \\ D_y = \begin{vmatrix} 2m & 3 \\ 1 & 3 \end{vmatrix} = 6m - 3 = 3(2m-1) \end{cases}$$

et $m = -\frac{1}{6}$ et $D=0$ et $D_x \neq 0$ et $D_y = -\frac{10}{3} \neq 0$
et $(x, y) \in \emptyset$

ou
 $m \neq -\frac{1}{6}$ et $(x, y) \in \left\{ \left(\frac{-12}{6m+1} ; \frac{-3(2m-1)}{6m+1} \right) \right\}$

$$3) \begin{cases} 4x + 2y = m \\ 3x - y = 1 \end{cases} \text{ et } (x, y) \in \mathbb{R} \times \mathbb{R}$$

$$\Leftrightarrow D = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = -10 \neq 0 \quad \text{et } D_x = \begin{vmatrix} m & 2 \\ 1 & -1 \end{vmatrix} = -m - 2$$

$$\text{et } D_y = \begin{vmatrix} 4 & m \\ 3 & 1 \end{vmatrix} = 4 - 3m$$

$$\text{et } (x, y) \in \left\{ \left(\frac{m+2}{10} ; \frac{3m-4}{10} \right) \right\} \quad (\forall m \in \mathbb{R}, D \neq 0)$$

$$4) \begin{cases} (m+2)x + y = 2 \\ (m+2)y + x = 4 \end{cases} \text{ et } (x, y) \in \mathbb{R} \times \mathbb{R} \quad \Leftrightarrow \begin{cases} (m+2)x + y = 2 \\ x + (m+2)y = 4 \end{cases}$$

$$\Leftrightarrow D = \begin{vmatrix} m+2 & 1 \\ 1 & m+2 \end{vmatrix} = (m+2)^2 - 1 = (m+2-1)(m+2+1) = (m+1)(m+3)$$

$$D_x = \begin{vmatrix} 2 & 1 \\ 4 & m+2 \end{vmatrix} = 2m + 4 - 4 = 2m$$

$$D_y = \begin{vmatrix} m+2 & 2 \\ 1 & 4 \end{vmatrix} = 4m + 8 - 2 = 4m + 6 = 2(2m + 3)$$

$$\text{et } m = -1 \text{ et } D = 0 \text{ et } D_x = -2 \neq 0 \text{ et } D_y = 2 \neq 0 \text{ et } (x, y) \in \emptyset$$

$$\text{ou } m = -3 \text{ et } D = 0 \text{ et } D_x = -6 \neq 0 \text{ et } D_y = -6 \neq 0 \text{ et } (x, y) \in \emptyset$$

$$\text{ou } m \notin \{-1, -3\} \text{ et } (x, y) \in \left\{ \left(\frac{2m}{(m+1)(m+3)} ; \frac{2(2m+3)}{(m+1)(m+3)} \right) \right\}$$

$$5) \begin{cases} (m^2+1)x + y = 0 \\ 2x + 5y = 1 \end{cases} \text{ et } (x,y) \in \mathbb{R} \times \mathbb{R}$$

$$\Leftrightarrow D = \begin{vmatrix} m^2+1 & 1 \\ 2 & 5 \end{vmatrix} = 5m^2 + 5 - 2 = 5m^2 + 3 (\neq 0)$$

$$D_x = \begin{vmatrix} 0 & 1 \\ 1 & 5 \end{vmatrix} = -1 \quad \text{et } (x,y) \in \left\{ \left(\frac{-1}{5m^2+3}, \frac{m^2+1}{5m^2+3} \right) \right\}$$

$$D_y = \begin{vmatrix} m^2+1 & 0 \\ 2 & 1 \end{vmatrix} = m^2+1$$

$$6) \begin{cases} x + my = 0 \\ 2x - 3y = 0 \end{cases} \text{ et } (x,y) \in \mathbb{R} \times \mathbb{R}$$

$$\Leftrightarrow D = \begin{vmatrix} 1 & m \\ 2 & -3 \end{vmatrix} = -3 - 2m$$

$$D_x = \begin{vmatrix} 0 & m \\ 0 & -3 \end{vmatrix} = 0 \quad \text{et } D_y = \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0$$

$$\text{et } m = -\frac{3}{2} \text{ et } D = 0 = D_x = D_y \quad \text{et } \begin{cases} x - \frac{3}{2}y = 0 \\ 2x - 3y = 0 \end{cases}$$

$$(\text{ et } (x,y) \in \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 2x - 3y = 0\}) \text{ juste mais peu académique}$$

$$\text{et } (x,y) \in \left\{ \left(x, \frac{2}{3}x \right) \mid x \in \mathbb{R} \right\}$$

$$\text{et } (x,y) \in \left\{ \left(\frac{3}{2}y, y \right) \mid y \in \mathbb{R} \right\}$$

$$\text{et } (x,y) \in \left\{ \left(k, \frac{2}{3}k \right) \mid k \in \mathbb{R} \right\}$$

$$\text{et } (x,y) \in \left\{ \left(\frac{3}{2}k, k \right) \mid k \in \mathbb{R} \right\}$$

à rédiger,
à choisir !

$$\text{ou } m \neq -\frac{3}{2} \text{ et } (x,y) \in \{(0,0)\}$$

14 Résoudre les systèmes paramétriques dans $\mathbb{R}^2$

$$\textcircled{1} \begin{cases} (m-4)x + my = -2 \\ 3x + y = 3 \end{cases}$$

$$2 \begin{cases} mx + 3y = 5 \\ 6x + 2y = 3 \end{cases}$$

$$3 \begin{cases} 7x - (m+5)y = 0 \\ 2x + y = 1 \end{cases}$$

$$4 \begin{cases} 4x + my = 3 \\ mx + 4y = m+1 \end{cases}$$

$$5 \begin{cases} ax + by = ab+1 \\ abx + ay = a^2+b \end{cases}$$

$$6 \begin{cases} x - (m+1)y = m \\ (m+2)x + (m+1)y = -1 \end{cases}$$

$$7 \begin{cases} (a+b)x + by = a \\ (a+b)x + ay = b \end{cases}$$

$$8 \begin{cases} y = mx + 2m \\ 2x = y - 3m \end{cases}$$

$$\textcircled{9} \begin{cases} 2x = my + m \\ 3x + 2y = 1 \end{cases}$$

$$\textcircled{1} \begin{cases} (m-4)x + my = -2 \\ 3x + y = 3 \end{cases}$$

$$\Leftrightarrow \begin{cases} D = \begin{vmatrix} m-4 & m \\ 3 & 1 \end{vmatrix} = m-4-3m = -2m-4 = (-2)(m+2) \\ D_x = \begin{vmatrix} -2 & m \\ 3 & 1 \end{vmatrix} = -2-3m = -3m-2 \\ D_y = \begin{vmatrix} m-4 & -2 \\ 3 & 3 \end{vmatrix} = 3 \begin{vmatrix} m-4 & -2 \\ 1 & 1 \end{vmatrix} = 3(m-2) \\ \text{et } m = -2 \text{ et } D = 0 \text{ et } D_x = 4 \neq 0 \text{ et } (x; y) \in \emptyset \\ \text{ou} \\ m \neq -2 \text{ et } D \neq 0 \text{ et } (x; y) \in \left\{ \left(\frac{+3m+2}{+2(m+2)}, \frac{-3(m-2)}{+2(m+2)} \right) \right\} \end{cases}$$

$$\textcircled{9} \begin{cases} 2x &= my + m \\ 3x + 2y &= 1 \end{cases} \Leftrightarrow \begin{cases} 2x - my = m \\ 3x + 2y = 1 \end{cases}$$

$$\Leftrightarrow D = \begin{vmatrix} 2 & -m \\ 3 & 2 \end{vmatrix} = 4 + 3m$$

$$D_x = \begin{vmatrix} m & -m \\ 1 & 2 \end{vmatrix} = 2m + m = 3m$$

$$D_y = \begin{vmatrix} 2 & m \\ 3 & 1 \end{vmatrix} = 2 - 3m$$

$$\text{et } m = -\frac{4}{3} \text{ et } D = 0 \text{ et } D_x = -4 \neq 0 \text{ et } (x; y) \in \emptyset$$

$$\text{ou } m \neq -\frac{4}{3} \text{ et } D \neq 0 \text{ et } (x; y) \in \left\{ \left(\frac{3m}{3m+4} ; \frac{-3m+2}{3m+4} \right) \right\}$$