

Interrogation de mathématique - 3

(Chapitre 1- 2)

a) Résoudre et discuter les équations suivantes :

1) $a(x - a^2) = b(x - b^2)$

2) $\frac{x}{m-1} + \frac{x}{m+1} = \frac{1}{m^2-1}$

b) Résoudre les équations suivantes $f(x) = 0$:

1) $f(x) = 3x^2 - 21x - 132$

5) $f(x) = \frac{2x+1}{x+3} - \frac{x+1}{x-4} - \frac{5(1-2x)}{x^2-x-12}$

2) $f(x) = x^4 - 2x^3 + 2x - 1$

6) $f(x) = (x-3)(x^2-6) - (3-x)(2x+3) + (4x-12)$

3) $f(x) = x^4 + 4x^3 + 6x^2 + 5x + 2$

7) $f(x) = -6x^2 - x + 15$

4) $f(x) = x^4 - 18x^2 + 81 = 0$

8) $f(x) = \frac{2x}{x+1} - \frac{3x}{x-1} + \frac{6}{1-x^2}$

7 a) 1) $a(x-a^2) = b(x-b^2)$ et $x \in \mathbb{R}$

$$\Leftrightarrow ax - a^3 = bx - b^3$$

$$\Leftrightarrow (a-b)x = a^3 - b^3$$

$$\Leftrightarrow (a-b)x = (a-b)(a^2+ab+b^2)$$

$$\Leftrightarrow a=b \text{ et } 0 \cdot x = 0 \text{ et } x \in \mathbb{R}$$

ou

$$a \neq b \text{ et } x = \frac{(a-b)(a^2+ab+b^2)}{(a-b)} \text{ et } x \in \{a^2+ab+b^2\}$$

2) $\frac{x}{m-1} + \frac{x}{m+1} = \frac{1}{m^2-1}$ et $x \in \mathbb{R}$

$$\Leftrightarrow \begin{cases} m=1 \text{ ou } m=-1 & \text{et } x \in \emptyset \end{cases}$$

ou

$$\begin{cases} m \notin \{1; -1\} & \text{et } \frac{x(m+1) + x(m-1)}{m^2-1} = \frac{1}{m^2-1} \end{cases}$$

$$\text{et } [(m+1) + (m-1)]x = 1$$

$$\text{et } 2mx = 1$$

$$\text{et } \begin{cases} m=0 & \text{et } 0 \cdot x = 1 \text{ et } x \in \emptyset \end{cases}$$

$$\text{ou } \begin{cases} m \neq 0 & \text{et } x = \frac{1}{2m} \text{ et } x \in \left\{ \frac{1}{2m} \right\} \end{cases}$$

$$\Leftrightarrow \begin{cases} m \in \{1; -1; 0\} & \text{et } x \in \emptyset \end{cases}$$

ou

$$\begin{cases} m \notin \{1; -1; 0\} & \text{et } x \in \left\{ \frac{1}{2m} \right\} \end{cases}$$



e) 1) $f(x) = 3x^2 - 21x - 132 = 0$ et $x \in \mathbb{R}$

$\Rightarrow x^2 - 7x - 44 = 0 \quad :3$

$\Rightarrow (x-11)(x+4) = 0$

$\Rightarrow x \in \{11; -4\}$

$$\begin{cases} m+n = -7 \\ m \cdot n = -44 \end{cases}$$

$$\Leftrightarrow \begin{cases} m = -11 \\ n = +4 \end{cases}$$

2) $f(x) = x^4 - 2x^3 + 2x - 1 = 0$ et $x \in \mathbb{R}$

$\Rightarrow (x-1) \underbrace{(x^3 - x^2 - x + 1)}_{g(x)} = 0$

$f(1) = 1 - 2 + 2 - 1 = 0$

$\Rightarrow (x-1)^2 \overbrace{(x-1)}^{g(x)} = 0$

$\Rightarrow (x-1)^3 \overbrace{(x+1)}^{g(x)} = 0$

$\Rightarrow (x-1) \overbrace{(x+1)}^{g(x)} = 0$

$\Rightarrow x \in \{1; -1\}$

$$\begin{array}{cccc|c} & & & & 1 \\ & 1 & -2 & 0 & 2 & -1 \\ & & 1 & -1 & -1 & 1 \\ \hline 1 & 1 & -1 & -1 & 1 & 0 \end{array}$$

et $g(1) = 1 - 1 - 1 + 1 = 0$

$$\begin{array}{cccc|c} & & & & 1 \\ & 1 & -1 & -1 & 1 & 1 \\ & & 1 & 0 & 1 & -1 \\ \hline 1 & 1 & 0 & -1 & 1 & 0 \end{array}$$

autre méthode:

$x^4 - 2x^3 + 2x - 1 = 0$ et $x \in \mathbb{R}$

$\Rightarrow (x^4 - 1) - (2x^3 - 2x) = 0$

$\Rightarrow (x^2 - 1)(x^2 + 1) - 2x(x^2 - 1) = 0$

$\Rightarrow (x^2 - 1) \overbrace{[x^2 + 1 - 2x]}^{g(x)} = 0$

$\Rightarrow (x^2 - 1) \overbrace{(x^2 - 2x + 1)}^{g(x)} = 0$

$\Rightarrow (x-1)(x+1)(x-1)^2 = 0$

$\Rightarrow (x-1)^3 \overbrace{(x+1)}^{g(x)} = 0$

$\Rightarrow x \in \{1; -1\}$



$$L) 3) f(x) = x^4 + 4x^3 + 6x^2 + 5x + 2 = 0 \quad \text{et } x \in \mathbb{R}$$

$$\Leftrightarrow (x+1) \underbrace{(x^3 + 3x^2 + 3x + 2)}_{g(x)} = 0$$

$$\Leftrightarrow (x+1)(x+2)(x^2 + x + 1) = 0$$

$$\text{or } x^2 + x + 1 = 0$$

$$\Leftrightarrow \Delta = 1^2 - 4 \cdot 1 \cdot 1 = -3 < 0$$

$$\text{et } x \in \emptyset$$

$$\begin{cases} m+n=1 \\ m \cdot n=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} m=? \\ n=? \end{cases}$$

$$\text{On a: } f(1) = 1 + 4 + 6 + 5 + 2$$

$$= 18 \neq 0$$

$$f(-1) = 1 - 4 + 6 - 5 + 2$$

$$= 0$$

$$\begin{array}{c|cccccc} & 1 & 4 & 6 & 5 & 1 & 2 \\ & & -1 & -3 & -3 & 1 & -2 \\ \hline -1 & 1 & 3 & 3 & 2 & 1 & 0 \end{array}$$

$$\text{et } g(-1) = -1 + 3 - 3 + 2 = 1 \neq 0$$

$$g(-2) = -8 + 12 - 6 + 2 = 0$$

$$\begin{array}{c|cccccc} & 1 & 3 & 3 & 1 & 2 \\ & & -2 & -2 & 1 & -2 \\ \hline -2 & 1 & 1 & 1 & 1 & 0 \end{array}$$

$$\Leftrightarrow (x+1)(x+2) = 0$$

$$\Leftrightarrow x \in \{-1; -2\}$$

$$4) f(x) = x^4 - 18x^2 + 81 = 0$$

$$\Leftrightarrow (x^2 - 9)^2 = 0$$

$$\Leftrightarrow x^2 - 9 = 0$$

$$\Leftrightarrow (x-3)(x+3) = 0$$

$$\Leftrightarrow x \in \{3; -3\}$$

on trouve :

$$\begin{cases} m+n=-18 \\ m \cdot n=81 \end{cases} \Leftrightarrow \begin{cases} m=-9 \\ n=-9 \end{cases}$$

$$5) \frac{2x+1}{x+3} - \frac{x+1}{x-4} - \frac{5(1-2x)}{x^2-x-12} = 0 \quad \text{et } x \in \mathbb{R} - \{-3; 4\}$$

$$\Leftrightarrow (2x+1)(x-4) - (x+1)(x+3) - 5(1-2x) = 0$$

$$\Leftrightarrow 2x^2 - 8x + x - 4 - x^2 - 4x - 3 - 5 + 10x = 0$$

$$\Leftrightarrow x^2 - x - 12 = 0$$

$$\Leftrightarrow (x-4)(x+3) = 0$$

$$\Leftrightarrow x \in \{4; -3\} \cap \mathbb{R} \setminus \{-3; 4\} \Leftrightarrow x \in \emptyset$$

$$\text{car } x^2 - x - 12 = (x-4)(x+3)$$

$$\begin{aligned}
 \text{e) 6) } & \frac{(x-3)}{(x^2-6)} - \frac{(3-x)}{(2x+3)} + \frac{(4x-12)}{(x-3)} = 0 \text{ et } x \in \mathbb{R} \\
 \Rightarrow & \frac{(x-3)}{(x^2-6)} - \frac{(-1)(x-3)}{(2x+3)} + 4 \frac{(x-3)}{(x-3)} = 0 \\
 \Rightarrow & \frac{(x-3)}{(x^2-6)} + \frac{(2x+3)}{(x-3)} + 4 = 0 \\
 \Rightarrow & \frac{(x-3)}{(x^2+2x+1)} = 0 \\
 \Rightarrow & \frac{(x-3)}{(x+1)^2} = 0 \\
 \Rightarrow & x \in \{3; -1\}
 \end{aligned}$$

$$\begin{aligned}
 7) & -6x^2 - x + 15 = 0 \text{ et } x \in \mathbb{R} \\
 \Rightarrow & 6x^2 + x - 15 = 0 \quad \leftarrow x(-1) \\
 \Rightarrow & 6x^2 + 10x - 9x - 15 = 0 \\
 \Rightarrow & 2x(3x+5) - 3(3x+5) = 0 \\
 \Rightarrow & (3x+5)(2x-3) = 0 \\
 \Rightarrow & x \in \left\{ -\frac{5}{3}; \frac{3}{2} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & \begin{cases} m+n = 1 \\ m \cdot n = 6 \cdot (-15) = -90 \end{cases} \\
 \Rightarrow & \begin{cases} m = +10 \\ n = -9 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 8) & \frac{2x}{x+1} - \frac{3x}{x-1} + \frac{6}{1-x^2} = 0 \text{ et } x \in \mathbb{R} - \{-1; 1\} \\
 \Rightarrow & \frac{2x}{x+1} - \frac{3x}{x-1} + \frac{-6}{x^2-1} = 0 \\
 \Rightarrow & 2x(x-1) - 3x(x+1) - 6 = 0 \\
 \Rightarrow & 2x^2 - 2x - 3x^2 - 3x - 6 = 0 \\
 \Rightarrow & -x^2 - 5x - 6 = 0 \\
 \Rightarrow & x^2 + 5x + 6 = 0 \Leftrightarrow (x+2)(x+3) = 0 \\
 \Rightarrow & x \in \{-2; -3\} \cap \mathbb{D} \\
 \Rightarrow & x \in \{-2; -3\}
 \end{aligned}$$

$$\begin{aligned}
 & \text{car } 1-x^2 = (1-x)(1+x) \\
 & \text{car } (1-x^2) = -(x^2-1)
 \end{aligned}$$

$$\begin{aligned}
 & \begin{cases} m+n = +5 \\ m \cdot n = +6 \end{cases} \Rightarrow \begin{cases} m = 2 \\ n = 3 \end{cases}
 \end{aligned}$$