Interrogation de mathématique - 4

(Algèbre – chapitre 2)

1) Simplifier les racines suivantes :

a) $\sqrt{x^6}$ b) $\sqrt{4x^2 - 44x + 121}$

c) $\sqrt{x^3 - 3x^2 + 3x - 1}$ d) $\sqrt{x^8}$

2) Effectuer les opérations suivantes :

a) $\left(\sqrt{48} + 5\sqrt{3}\right)^2$

b) $\dfrac{4}{\sqrt{7} - \sqrt{5}} + \dfrac{1}{\sqrt{6} - \sqrt{7}} + \dfrac{6}{\sqrt{7} - 5}$

3) Rationaliser le dénominateur des fractions suivantes :

a) $\dfrac{21}{5\sqrt{28}}$ b) $\dfrac{\sqrt{18}}{\sqrt{2} - \sqrt{5} + \sqrt{3}}$ c) $\dfrac{\sqrt{3+2\sqrt{2}}}{\sqrt{3-2\sqrt{2}}}$

1). a) $\sqrt{x^6} = \sqrt{(x^3)^2} = |x^3|$

b) $\sqrt{4x^2-44x+121} = \sqrt{(2x-11)^2} = |2x-11|$

c) $\sqrt{x^3-3x^2+3x-1} = \sqrt{(x-1)^3} = |x-1|\sqrt{x-1}$

$= (x-1)\sqrt{x-1}$

car $(x-1) \geq 0$

d) $\sqrt{x^8} = \sqrt{(x^4)^2} = |x^4| = x^4$ car $x^4 \geq 0$

2). a) $(\sqrt{48} + 5\sqrt{3})^2 = (4\sqrt{3} + 5\sqrt{3})^2 = (9\sqrt{3})^2 = 81 \cdot 3 = 243$

b) $\frac{4}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{6}-\sqrt{7}} + \frac{6}{\sqrt{7}-5} = \frac{\overset{2}{\cancel{4}}(\sqrt{7}+\sqrt{5})}{\cancel{2}} + \frac{\sqrt{6}+\sqrt{7}}{(-1)} + \frac{\cancel{6}(\sqrt{7}+5)}{-\cancel{18}-3}$

$= \frac{6\sqrt{7}+6\sqrt{5}-3\sqrt{6}-3\sqrt{7}-\sqrt{7}-5}{3} = \frac{2\sqrt{7}+6\sqrt{5}-3\sqrt{6}-5}{3}$

c) $5\sqrt{12} - 2\sqrt{\frac{3}{4}} + 2\sqrt{27} - 8\sqrt{\frac{3}{16}} =$

$10\sqrt{3} - \cancel{2}\frac{\sqrt{3}}{\cancel{2}} + 6\sqrt{3} - \overset{2}{\cancel{8}}\frac{\sqrt{3}}{\cancel{4}} =$

$(10 - 1 + 6 - 2)\sqrt{3} = 13\sqrt{2}$

d) $a\sqrt{\frac{a-b}{a+b}} + b\sqrt{\frac{a+b}{a-b}} =$

$\frac{a\sqrt{a-b}}{\sqrt{a+b}} + \frac{b\sqrt{a+b}}{\sqrt{a-b}} =$

$\frac{a\sqrt{a-b}\cdot\sqrt{a+b}}{a+b} + \frac{b\sqrt{a+b}\cdot\sqrt{a-b}}{a-b} =$

$\frac{a\sqrt{a^2-b^2}(a-b) + b\sqrt{a^2-b^2}\cdot(a+b)}{(a+b)(a-b)} =$

$\frac{\sqrt{a^2-b^2}\,[a^2 \cancel{-ab} \cancel{+ab} + b^2]}{(a+b)(a-b)} = \frac{\sqrt{a^2-b^2}\,(a^2+b^2)}{a^2-b^2}$

3) a) $\dfrac{21}{5\sqrt{28}} = \dfrac{21}{5\cdot 2\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\overset{3}{\cancel{21}}\cdot\sqrt{7}}{10\cdot\cancel{7}} = \dfrac{3\sqrt{7}}{10}$

b) $\dfrac{\sqrt{18}}{\sqrt{2}-\sqrt{5}+\sqrt{3}} = \dfrac{3\sqrt{2}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}} \cdot \dfrac{(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(\sqrt{2}+\sqrt{3})+\sqrt{5}} = \dfrac{3\sqrt{2}\,(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$= \dfrac{3\sqrt{2}\,(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(\cancel{2}+2\sqrt{2}\cdot\sqrt{3}+\cancel{3})-\cancel{5}} = \dfrac{3\cancel{\sqrt{2}}\cdot(\sqrt{2}+\sqrt{3}+\sqrt{5})}{2\cancel{\sqrt{2}}\cdot\sqrt{3}} = \dfrac{\sqrt{3}\cdot\cancel{\sqrt{3}}(\sqrt{2}+\sqrt{3}+\sqrt{5})}{2\cancel{\sqrt{3}}}$

$= \dfrac{\sqrt{3}\,(\sqrt{2}+\sqrt{3}+\sqrt{5})}{2}$

c) $\dfrac{\sqrt{3+2\sqrt{2}}}{\sqrt{3-2\sqrt{2}}} = \dfrac{\sqrt{3+2\sqrt{2}}}{\sqrt{3-2\sqrt{2}}} \cdot \dfrac{\sqrt{3+2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}} = \dfrac{3+2\sqrt{2}}{\sqrt{9-8}} = 3+2\sqrt{2}$

or

$\dfrac{\sqrt{3+2\sqrt{2}}}{\sqrt{3-2\sqrt{2}}} = \sqrt{\dfrac{3+2\sqrt{2}}{3-2\sqrt{2}} \cdot \dfrac{3+2\sqrt{2}}{3+2\sqrt{2}}} = \sqrt{\dfrac{(3+2\sqrt{2})^2}{9-8}} = 3+2\sqrt{2}$