

Interrogation de mathématique - 4

(Algèbre – chapitre 2)

1) Simplifier les racines suivantes :

a) $\sqrt{x^4}$

b) $\sqrt{9x^2 - 30x + 25}$

c) $\sqrt{x^4 + 4x^3 + 6x^2 + 4x + 1}$

d) $\sqrt{x^2}$

e) $\sqrt{x^3}$

f) $\sqrt{x^7}$

2) Effectuer les opérations suivantes :

a) $(\sqrt{2} - \sqrt{5})^2$

b) $(\sqrt{8} + 3\sqrt{2})^2$

c) $(\sqrt{x} + 5)^2$

d) $(2\sqrt{15} + \sqrt{3}) \cdot (2\sqrt{15} - \sqrt{3})$

3) Rationaliser le dénominateur des fractions suivantes :

a) $\frac{4}{\sqrt{8}}$

b) $\frac{5}{3\sqrt{20}}$

c) $\frac{8}{\sqrt{5} - \sqrt{3}}$

d) $\frac{2}{\sqrt{3} + \sqrt{5} + \sqrt{10}}$

e) $\frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}}$

4) Effectuer les opérations suivantes :

a) $\frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{2} - 1}$

b) $\frac{1}{\sqrt{3} - \sqrt{2}} - \frac{1}{3 - \sqrt{2}} + \frac{5}{2 - \sqrt{3}}$

$$1) a) \sqrt{x^4} = \sqrt{(x^2)^2} = |x^2| = x^2 \text{ (car } x^2 \geq 0)$$

$$b) \sqrt{9x^2 - 30x + 25} = \sqrt{(3x-5)^2} = |3x-5|$$

$$c) \sqrt{x^4 + 4x^3 + 6x^2 + 4x + 1} = \sqrt{(x+1)^4} = |(x+1)^2| = (x+1)^2$$

$$d) \sqrt{x^2} = |x|$$

$$e) \sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2} \cdot \sqrt{x} = |x| \cdot \sqrt{x} = x \cdot \sqrt{x} \text{ (car } x \geq 0)$$

et $x \in \mathbb{R}_+$

$$f) \sqrt{x^7} = \sqrt{x^6 \cdot x} = \sqrt{x^6} \cdot \sqrt{x} = \sqrt{(x^3)^2} \cdot \sqrt{x} = |x^3| \cdot \sqrt{x} = x^3 \cdot \sqrt{x} \text{ (car } x \geq 0)$$

et $x \in \mathbb{R}_+$

$$2) a) (\sqrt{2} - \sqrt{5})^2 = 2 - 2\sqrt{2} \cdot \sqrt{5} + 5 = 7 - 2\sqrt{10}$$

$$b) (\sqrt{8} + 3\sqrt{2})^2 = (2\sqrt{2} + 3\sqrt{2})^2 = (5\sqrt{2})^2 = 25 \cdot 2 = 50$$

$$c) (\sqrt{x} + 5)^2 = x + 2\sqrt{x} \cdot 5 + 25 = x + 10\sqrt{x} + 25$$

$$d) (2\sqrt{15} + \sqrt{3}) \cdot (2\sqrt{15} - \sqrt{3}) = (2\sqrt{15})^2 - (\sqrt{3})^2 = 4 \cdot 15 - 3 = 60 - 3 = 57$$

$$3) a) \frac{4}{\sqrt{8}} = \frac{24}{2\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\text{ou } \frac{4}{\sqrt{8}} = \frac{\sqrt{16}}{\sqrt{8}} = \sqrt{\frac{16}{8}} = \sqrt{2}$$

$$b) \frac{5}{3\sqrt{20}} = \frac{5}{3 \cdot 2\sqrt{5}} = \frac{\sqrt{5} \cdot \sqrt{5}}{6 \cdot \sqrt{5}} = \frac{\sqrt{5}}{6}$$

$$c) \frac{8}{\sqrt{5} - \sqrt{3}} = \frac{8}{\sqrt{5} - \sqrt{3}} \cdot \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{8(\sqrt{5} + \sqrt{3})}{5 - 3} = \frac{8(\sqrt{5} + \sqrt{3})}{2} = 4(\sqrt{5} + \sqrt{3})$$

$$d) \frac{2}{\sqrt{3} + \sqrt{5} + \sqrt{10}} = \frac{2}{(\sqrt{3} + \sqrt{10}) + \sqrt{5}} \cdot \frac{\sqrt{3} + \sqrt{10} - \sqrt{5}}{\sqrt{3} + \sqrt{10} - \sqrt{5}} = \frac{2(\sqrt{3} + \sqrt{10} - \sqrt{5})}{(\sqrt{3} + \sqrt{10})^2 - (\sqrt{5})^2}$$

$$= \frac{2(\sqrt{3} + \sqrt{10} - \sqrt{5})}{(3 + 2\sqrt{30} + 10) - 5} = \frac{2(\sqrt{3} + \sqrt{10} - \sqrt{5})}{8 + 2\sqrt{30}} = \frac{2(\sqrt{3} + \sqrt{10} - \sqrt{5})}{2(4 + \sqrt{30})}$$

$$= \frac{(\sqrt{3} + \sqrt{10} - \sqrt{5}) \cdot (4 - \sqrt{30})}{16 - 30} = \frac{(\sqrt{3} + \sqrt{10} - \sqrt{5})(4 - \sqrt{30})}{-14}$$

$$\begin{aligned}
 e) \quad \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} &= \frac{\sqrt{3-\sqrt{5}} \cdot \sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}} \cdot \sqrt{3-\sqrt{5}}} = \frac{3-\sqrt{5}}{\sqrt{(3+\sqrt{5})(3-\sqrt{5})}} \\
 &= \frac{3-\sqrt{5}}{\sqrt{9-5}} = \frac{3-\sqrt{5}}{2}
 \end{aligned}$$

$$\text{or} \quad \frac{\sqrt{3-\sqrt{5}}}{\sqrt{3+\sqrt{5}}} = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{|3-\sqrt{5}|}{\sqrt{4}} = \frac{3-\sqrt{5}}{2}$$

$$4) a) \quad \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} + \frac{\sqrt{2}+1}{1} = 2\sqrt{2}$$

$$\begin{aligned}
 e) \quad \frac{1}{\sqrt{3}-\sqrt{2}} - \frac{1}{3-\sqrt{2}} + \frac{5}{2-\sqrt{3}} &= \frac{\sqrt{3}+\sqrt{2}}{1} - \frac{3+\sqrt{2}}{7} + \frac{5(2+\sqrt{3})}{1} \\
 &= \frac{7\sqrt{3}+7\sqrt{2}-3-\sqrt{2}+10+35\sqrt{3}}{7} = \frac{42\sqrt{3}+6\sqrt{2}+67}{7}
 \end{aligned}$$